# Certainty and Severity of Sanctions: An Uncertain Tradeoff*

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**Abstract**

Deterrence theorists have long known that the certainty and severity of sanctions are substitutes, so the analysis has focused on identifying the optimal tradeoff between the two. Yet many enforcement environments feature rewards as well as sanctions; and many environments are neither mandatory nor universal as the standard model assumes. We show that introducing rewards and optional participation into the basic deterrence model significantly complicates the certainty-severity tradeoff. If rewards are present and participation is universal, adjusting the probability of a sanction has a greater deterrent effect than adjusting its magnitude. Still, for a given change in probability it is possible to change the magnitude in order to keep the deterrence incentives unchanged. If, however, rewards are present and participation is optional, the probability-magnitude tradeoff becomes more complex still, and it is no longer possible to compensate for a change in one of these two variables by adjusting the other. In some cases — which we will identify in the analysis — probability and magnitude have opposite effects on all outcomes (compliance, violation and participation). We emphasize that policymakers face significant uncertainty about the precise relationship between the probability and magnitude of sanctions given the challenges of recognizing the presence of rewards, evaluating their magnitude, and modifying their size.

Keywords: deterrence, compliance, participation, sanctions, rewards, carrots, sticks, multiplier
JEL codes: K10, K20, K42

## 1 Introduction

Scholars of deterrence are keenly aware that the certainty and severity of punishment are closely linked. So is everyone else. A motorist would disregard a speed limit if she knows that police never monitors the road she is driving on. A child would ignore a parent's empty threats. What sets deterrence scholars apart is that they understand the exact relationship between the two enforcement instruments. At least since 1968 when Gary Becker published his famous article (Becker, 1968), scholars have known that the probability and magnitude of sanctions are substitutes. It is their product that determines the expected sanction, and it is the expected sanction that controls

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the behavior of rational actors. The task of the optimal deterrence theory has been finding the optimal tradeoff between the probability and magnitude of sanctions given their interchangeability (Polinsky and Shavell, 1979).

This paper explains why in a large set of cases the tradeoff between the certainty and severity of sanctions is significantly less clear-cut than is generally understood. A policymaker interested in adjusting the certainty of sanctions to compensate for a change in their severity (or vice versa) faces uncertainty about both the precise relationship between the two variables and the possibility of maintaining a given level of deterrence by adjusting these variables alone. The complexity arises for two reasons. First, in many settings where law enforcers and deterrence theorists see only nominal sanctions and detection uncertainty, there are often rewards as well. Second, in many settings where law enforcers and deterrence theorists see only a choice between compliance and violation, there is often an alternative of abstaining from participation in the regulatory regime altogether. Introducing rewards and optional participation into the basic analysis of deterrence greatly complicates the probability-magnitude tradeoff and, in some case, reverses the results of the traditional model: probability and magnitude may have the *opposite* effect on all behavioral outcomes of the enforcement scheme, that is, on compliance, violations and participation.

To see the real-world significance of rewards and optional participation, consider a real estate developer preparing to sell land lots for future construction of single-family homes. The developer needs to decide whether to report his future profits as a low-taxed capital gain or as a high-taxed ordinary income (IRC sec. 1(a), (h)). The developer knows that if he sells lots of undeveloped land, tax law would view him as an investor, and his gains would be eligible for the low rate. If, however, he develops the lots by laying roads, water pipes, electricity, and so on, the tax law will view him as a dealer and tax his gain as ordinary income. Unfortunately, the distinction between a real estate investor and a dealer is notoriously vague. The legal analysis must proceed under a multi-factor test, some of the factors are themselves ambiguous, no single factor is controlling, and the answer is not determined by counting factors pointing in each direction (these are the so-called Winthrop factors). Moreover, the developer thinks that his profit would be higher if he develops the lots before selling them, so the choice of selling undeveloped land is costly to him.

Assume that the developer would like to report all his profits as capital gains, and he is ready to do what the law requires (though with some uncertainty) to be an investor for tax purposes. He knows that if he is audited, the IRS is likely to agree with the capital gain treatment (say with the probability of 75%), but there is a chance that it will not. If the IRS disagrees, the developer would owe extra tax plus a penalty. To decide whether acting as an investor is worth it, the developer discounts the sanction by the probability of its imposition (which for simplicity we will assume is the probability of audit times 0.75).

The deterrence theory posits that if the government wanted to induce more developers to take the low-tax but costly (in terms of foregone profits) position while actually acting like investors rather than dealers, the government could increase the statutory penalty, or the audit rate, or some combination of the two. Only their product matters to a developer who responds to expected, rather than nominal, sanctions. This is the calculation underlying the canonical deterrence model of Becker (1968) and, more specifically, the canonical tax evasion model of Allingham and Sandmo (1972).

But this scenario is incomplete, and so is the classical deterrence calculation just described. An increase in the statutory penalty is clearly bad news for the developer. An increase in the audit rate is not. The difference exists because in addition to sanctions and detection, there is a hidden reward in the picture. If the IRS audits the developer and agrees with the capital gain characterization, that determination will resolve the uncertainty not only in the current year, but in future years as well. As long as the developer does not significantly change the way he does business, future tax savings that were uncertain before the audit become certain after it. Given that the carrot of eliminating future tax uncertainty is available only to audited taxpayers, an increase in the audit rate is a mixed blessing. More frequent audits make both this carrot and the stick more likely. In contrast, higher nominal sanctions are nothing but trouble.

There is a further complication. The real estate developer is not obligated to stay in his business (or to enter it in the first place). If expected sanctions are high enough—whether because of high nominal sanctions, high uncertainty of their imposition, or both—the developer may choose to abstain from participating in the regime altogether. This participation decision is often overlooked in deterrence models. As far as we know, it has been ignored entirely in evaluating the certainty-severity tradeoff.

In this paper, we introduce rewards and the participation choice into the standard Becker (1968) model, and we derive novel results. First, if the standard model is expanded to incorporate rewards available only to audited agents (while assuming universal participation as is typical), certainty and severity of sanctions are no longer substitutes. Rather, the expected sanction is the product of the detection probability and the sum of the carrot and the stick. This result is hardly surprising. It has been long-known that carrots and sticks are substitutes in their incentive effects. We show elsewhere that this is true only if participation is universal, but that is what we assume here. Thus, it is the sum of a carrot and a stick—not the stick alone—that determines the overall incentive to comply. When detection is uncertain, rational agents discount that sum by the rate of detection.

We derive the relationship between an increase in the probability of detection and an increase in the sanction that would have an equivalent incentive effect. The latter increase, it turns out, must be larger than in a setting without carrots. This is so because increasing the probability of detection increases the total incentive to comply (the sum of a carrot and a stick). But raising the penalty increases only one component of that incentive. Necessarily, higher penalties are a less effective deterrent than higher probabilities where participation is universal and carrots exist alongside sticks.

Our second result comes from modifying the standard setting not only by introducing carrots but by incorporating optional participation as well. In this case the link between the magnitude and probability of sanctions weakens further. Optional participation means that the model needs to accommodate agents who decide not to participate. One simple (though by no means only) way of introducing such non-participants is to assume that all agents belong to two groups: one with high benefits from participation and another with low benefits. For high-benefit agents, participation brings positive payoffs whether they comply or violate the rule. These agents choose between compliance and violation. For low-benefit agents, we show that the payoff from participation is positive only if they comply. These agents choose between compliance and non-participation.

The simple relationship between the certainty and severity of sanctions breaks down in this setting because the three enforcement variables that we study (sanctions, rewards, and probabilities) affect high- and low-benefit agents differently. High-benefit agents respond to these variables as the standard theory predicts, as modified by our earlier point about carrots. For low-benefit agents, however, we show that carrots and sticks are complements rather than substitutes. Higher sticks reduce the number of compliers in the low-benefit group, and may also turn some high-benefit agents into low-benefit ones. Higher carrots increase the number of compliers in the low-benefit group and may also turn some low-benefit agents into high-benefit ones. And of course, the effect of carrots and sticks on behavior is discounted by the probability of detection.

These interactions create complexity. First, it is impossible in this setting to replicate a change in probability by adjusting only the magnitude of the sanction. In order to preserve incentives while changing the probability of detection the regulator must change the magnitude of both the sanction and the reward. So while the probability-magnitude tradeoff still exists, adjusting the two in the opposite direction is not sufficient to preserve deterrence.

Second, setting aside the goal of preserving incentives to comply, we show that changes in the magnitude and probability of sanctions may not even move these incentives in the same direction. If sanctions are larger than rewards, higher sanctions and greater probability of detection have similar deterrent effects. But if rewards are larger than sanctions, the same would not be true because increasing the probability would be similar to increasing the reward rather than the sanction. Overall, if rewards are present and participation is optional, the basic insights of the deterrence theory would lead law enforcers seriously astray.

The relevance (and importance) of our results depends on whether rewards and optional participation are widespread, especially by comparison to the settings with no rewards and universal participation that are assumed in the standard deterrence model. We believe that while our settings are not ubiquitous, they are highly realistic.

Tax law offers many examples similar to the one featuring the real estate developer. Tax incentives such as R&D tax credits, the depreciation deduction, the absence of permanent establishment in a foreign jurisdiction, tax ownership, tax characterization of financial instruments (including debt, equity, and various derivatives) all present taxpayers with the same choices that our model reflects. Tax law is by no means the only area where participation is optional, inspections / audits are uncertain, the law is vague, and inspections produce the benefit of resolving future legal uncertainty. Environmental regulations, health and safety regulation, financial regulation all fit the bill.

Moreover, the resolution of future legal uncertainty is not the only possible carrot. Returning to our initial example, assume that the developer is actually Developer LLP, a large firm taxed as a partnership. The firm has outside investors, major bank creditors, and it prepares annual financial statements. If a tax benefit such as the reduced capital gains rate is uncertain, the firm cannot reflect the benefit in its financial reports, and may need to increase its discretionary cash holdings as a result (Dyreng, Hanlon, and Maydew, 2019). Resolving this uncertainty may have an immediate financial benefit in addition to the advantage of eliminating uncertainty in future years. Financial accounting is only one example of a regulatory regime that may "reward" the firm with a carrot in the wake of a successful tax audit.

Furthermore, regulators are increasingly embracing risk-based regulation, which is now widespread in environmental, financial, food safety regulation, and even regulation of legal services (Black and Baldwin, 2012). If a firm deemed by a regulator as high-risk successfully completes an inspection (or several inspections in a row), the regulator is likely to reclassify the firm as low-risk. This change in status would lead to fewer costly inspections, lower likelihood of detection of future violations, and possibly even a lower schedule of sanctions should the regulator discover a violation. All of these are substantial benefits that serve as the carrot in our model. Finally, none of the regimes just discussed are mandatory or universal.

Our results highlight the challenges faced by real-world regulators. It may be plausible to think that regulators have the ability to control statutory sanctions and audit probabilities. But given the examples of rewards that we offer, it is clearly much more difficult for regulators to alter the certainty and severity of sanctions to account for the presence of rewards. In fact, it may be challenging for the regulator to figure out if the meaningful reward is even in the picture. Yet the regulator needs to know this in order to evaluate whether the basic certainty-severity tradeoff holds. The regulators needs to know not only the existence but also the magnitude of the reward in order to calculate the certainty-severity tradeoff even if participation is universal. And the regulator needs to both know the magnitude of the reward and be able to adjust it in order to fully control deterrence in optional participation regimes.

Regulators' ability to spot available rewards is uncertain; their ability to evaluate the size of these rewards is doubtful; and the regulators' ability to recognize the presence of rewards, evaluate their magnitude, and also adjust their size as we suggest is mostly implausible. So the practical takeaway from our investigation is that regulators administering regimes where rewards may be present and participation is optional should be careful with following the simple prescription of the deterrence theory, elegant as it may be. Adjusting the probability and magnitude of sanctions would not have the same effect on behavior of regulated parties. The certainty-severity tradeoff is not nearly as clear as what deterrence theorists believed it to be for a long time.

The plan for the remaining of this paper is as follows. In Section 2 we introduce our general deterrence model that incorporates carrots and optional participation. We demonstrate our results in a simplified version of the model, which captures all of the main insights, and resort to the Appendix for the general proofs. In Section 3 we consider the implications of the model for real-life regulatory settings and, in Section 4 we will conclude.

# 2 The model

## 2.1 Model setup

We consider a principal regulating an activity with optional participation and a population of agents described by the probability distribution $F(b,e)$ of the agents' two-dimensional types $(b,e)$. Agents derive a fixed benefit from participation $b$ (which can be positive or negative) and, if they participate, face a fixed cost of compliance $e > 0$. Agents make two decisions: they decide whether to participate and, if so, they decide whether to comply.[1] Agents are risk neutral and maximize their expected payoff.

The principal monitors agents with probability $0 < p \leq 1$, rewards compliance with a carrot $c > 0$ and punishes violations with a stick $s > 0$. Enforcement is imperfectly accurate: The principal correctly identifies compliance with probability $\frac{1}{2} < q_k < 1$ and hence may erroneously classify a complying agent's conduct as violation with the complementary probability $1 - q_k$ (which can be seen as "convicting the innocent"). Similarly, the principal correctly identifies violation with probability $\frac{1}{2} < q_v < 1$ and may erroneously classify a violating agent's conduct as compliance with probability $1 - q_v$ ("acquitting the guilty").[2]

As a result of imperfect monitoring, agents who are not monitored may receive a carrot with probability $\phi_c \geq 0$, a stick with probability $\phi_s \geq 0$, or neither of the two with the residual probability $1 - \phi_c - \phi_s \geq 0$.[3] To illustrate, if $\phi_c = 1$, then non-monitored agents receive a carrot with certainty.

[1]Non-participants earn no benefit and violators incur no cost.

[2]We focus on the plausible scenarios where the principal does better than a coin toss by assuming that both probabilities are greater than $\frac{1}{2}$.

[3]See Dari-Mattiacci et al. (2009) examining the incentive problems that arise from rewarding or punishing non-monitored agents.

This case corresponds to a subsidy given to agents who engage in the activity and are not found in violation of the rule, that is, the subsidy is taken away if the agent is monitored and found in violation. Similarly, if $\phi_s = 1$, then non-monitored agents are subject to a stick with certainty. This amounts to taxing the activity unless the agent is found in compliance. Finally, if $\phi_c = \phi_s = 0$, then carrots and sticks are applied only to monitored agents.

## 2.2 Simplifying assumptions

To make our point in the starkest possible way, we make here three simplifying assumptions. These assumptions will be relaxed in the Appendix.

**Assumption 1.** $q_k = q_v = q$ *(symmetric accuracy)*

Assumption 1 implies that the principal is equally accurate in detecting compliance and violations. Note that it follows from the general setup that $\frac{1}{2} < q < 1$.

**Assumption 2.** $\phi_c = \phi_s = 0$ *(conditional incentives)*

Assumption 2 implies that if an agent in not monitored, that is, with probability $1 - p$, he neither receives a carrot nor pays a stick, irrespective of the agent behavior.

**Assumption 3.** $F(b,e) = B[h] \times U[0,1]$ *(independent characteristics)*

Assumption 3 guarantees that the two characteristics that define an agent's type are independently distributed[4] and hence can be considered in turn. The benefit $b$ follows a Bernoulli distribution with probability $h$, which implies that the population of agents is divided into two groups: a portion $h$ of the agents derive a high benefit, $b_H$, from participation, the remaining portion $1 - h$ of the agents derive a low benefit, $b_L$, from participation.[5] In each of these two groups, the cost of compliance $e$ is uniformly distributed between 0 and 1.

**Assumption 4.** $b_H > p(qs - (1-q)c) \geq b_L$ *(baseline policy)*

Assumption 4 guarantees that, for a given baseline policy—consisting of a carrot $c$, a stick $s$, and a probability of monitoring $p$—all of the high-benefit agents will find it advantageous to participate, while some (or all) of the low-benefit agents refrain from participating. This provides a useful starting point for the analysis.

## 2.3 Analysis

Given the general setup in Section 2.1 and the simplifying assumptions in Section 2.2, an agent of type $(b,e)$ anticipates a payoff that depends on his choice whether to participate and, if so, whether to comply or violate. Non-participating agents earn a payoff equal to 0. If an agent participates but violates, he will earn a payoff $\Pi_v(b)$ calculated as follows. The agent will earn the benefit $b$ but will face enforcement with probability $p$. If monitored, the agent will pay a stick $s$ with probability $q$ (that is, if the principal correctly identifies the violation) but may also receive a carrot with probability $1 - q$ (that is, if the principal erroneously detects compliance). If the

[4] Note that this not necessarily the case in the general setup of Section 2.1 and, hence, in the Appendix.

[5] In a Bernoulli distribution the outcomes as 0 and 1, but the same structure can be used with different binary outcomes after a simple linear transformation.

agent participates and complies, he earns a payoff $\Pi_k(b,e)$: he earns the benefit $b$, receives a carrot with probability $pq$ (that is, if monitored and found in compliance), faces a stick with probability $p(1-q)$ (that is, if monitored and erroneously found in violation), and incurs costly effort equal to $e$. Table 1 summaries the agent's payoffs.

| Action | Payoff |
|---|---|
| The agent participates and complies | $\Pi_k(b,e) = b + pqc - p(1-q)s - e$ |
| The agent participates but violates | $\Pi_v(b) = b + p(1-q)c - pqs$ |
| The agent does not participate | 0 |

Table 1: Agent's payoffs

Assumption 4 guarantees that $\Pi_v(b_H) > 0$, which in turn gives high-benefit agents the option to earn a positive payoff by participating and violating the rule. Since violation dominates non-participation, high-benefit agents choose between compliance and violation depending on their costs of effort. In particular, a high-benefit agent will comply if the payoff from compliance is greater than the payoff from violation, that is, if $\Pi_k(b_H,e) > \Pi_v(b_H)$, which is the case if the agent's cost of effort is lower than the compliance threshold

$$e_k \equiv p(c+s)(2q-1) \quad (1)$$

and will violate otherwise. Given the uniformity assumption (Assumption 3), a portion $e_k$ of the high-benefit agents participate and comply, while the remaining portion $1-e_k$ participate and violate. Characteristically, none of them chooses to stay out of the regulated activity.

The second part of Assumption 4 guarantees that $\Pi_v(b_L) \leq 0$, which implies that non-participation dominates violation. Consequently, low-benefit agents choose between compliance and non-participation and opt for the former if $\Pi_k(b_L,e) > 0$, which is the case if the agent's cost of effort is lower than the participation threshold

$$e_p \equiv b_L + p(qc - (1-q)s) \quad (2)$$

Due again to the uniformity assumption, a portion $e_p$ of the low-benefit agents participate and comply, while the remaining portion $1-e_p$ refrain from participation. None of them violates the rule. Figure 1 below offers a visualization of the agents' behavior in a typical baseline setting satisfying Assumption 4.

Under a baseline policy satisfying Assumption 4, the two groups of agents behave in radically different ways. The behavior of high-benefit agents is controlled by the compliance threshold $e_k$, which, as it is easy to see, increases in $c$, $s$, and $p$. The following two lemmas show how the thresholds $e_k$ and $e_p$ that control the behavior of high- and low-benefit agents, respectively, react to changes in the magnitude or probability of sanctions. We start from the high-benefit agents, who fully participate: Lemma 1 generalizes Becker's fundamental insight that, for purposes of deterring undesirable behavior, one can equivalently increase the probability of monitoring or the magnitude of the sanction. Becker was concerned about sticks only, we prove his result in a framework with carrots and sticks.

**Lemma 1.** *Increasing the probability of apprehension, $p$, or the total magnitude of the sanctions, $c+s$, by the same multiplier results in the same increase in the compliance threshold, $e_k$, irrespective of the ratio $\frac{c}{s}$.*

*Proof.* Omitted. □

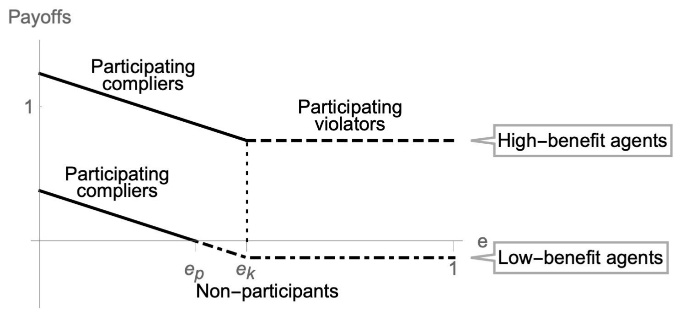


Figure 1: Agents' behavior under the baseline policy (Simulation values: $c = s = 1$, $p = \frac{1}{2}$, $q = \frac{3}{4}$, $b_H = 1$, $b_L = \frac{1}{8}$)

If, as in Becker (1968), the only policy instrument is the stick, then the same percent increase in $p$ or in $s$ results in the same increase in deterrence. However, when also carrots are employed, this mechanical rule no longer holds in its simplest form and one has to consider the *total* magnitude of the sanctions, that is, the sum of the carrots and the stick, $c + s$.

Focusing now on low-benefit agents, who do not fully participate, Lemma 2 shows that, for those agents, whose behavior is controlled by the participation threshold $e_p$, the relationship between the probability and magnitude of sanctions can be preserved only if the ratio between the carrot and the stick, $\frac{c}{s}$, remains constant. The intuition is that the carrot and the stick have opposite effects on the behavior of low-benefit agents; the probability of apprehension acts on both the application of the carrot and the application of the stick. Hence, a change in the probability $p$ can be counterbalanced only if both the carrot $c$ and the stick $s$ are accordingly adjusted.

**Lemma 2.** *Increasing the probability of apprehension, $p$, or the total magnitude of the sanctions, $c + s$, by the same multiplier results in the same increase in the participation threshold, $e_p$, if and only if the ratio $\frac{c}{s}$ remains constant.*

*Proof.* Let us first show that if the ratio $\frac{c}{s}$ remains constant, then increasing $p$ by a multiplier $x$ or increasing $c + s$ by the same multiplier $x$ results in the same level of the participation threshold $e_p$. Let $p' = xp$, then the participation threshold becomes $e'_p = b_L + xp\,(qc - (1 - q)\,s)$. Now take $c'$ and $s'$ such that $\frac{c'}{s'} = \frac{c}{s}$ and $c' + s' = x\,(c + s) = xc + xs$. It is easy to show that the ratio $\frac{c}{s}$ remains constant if and only if $c' = xc$ and $s' = xs$. Using this fact we can write $b_L + p\,(qc' - (1 - q)\,s') = b_L + p\,(qxc - (1 - q)\,xs) = e'_p$. Showing the opposite implication is trivial, that is, if increasing $p$ by a multiplier $x$ and increasing $c + s$ by the same multiplier $x$ results in the same level of the participation threshold $e_p$, then $\frac{c'}{s'} = \frac{c}{s}$. □

## 2.4 Results

In the Becker model, increasing the probability $p$ or the magnitude $s$ of the punishment by the same multiplier results in the same increase in deterrence. In our model with carrots and optional participation, we show that to replicate the effects of an increase in probability from $p$ to $xp$, where $x > 1$,[6] the principal needs to implement one of the following policies, depending on the situation:

[6]Obviously the reverse applies to $x < 1$, that is, to reductions in $p$.

1. If there are no carrots ($c = 0$): Increase the stick by multiplying it by $x$. In this case we have $e_k = ps(2q-1)$ and $e_p = b_L - ps(1-q)$ and hence $p$ and $s$ are perfect substitutes.

2. If there are carrots but all agents fully participate: Increase the *sum* $c + s$ by multiplying it by $x$. In this case the only relevant threshold is $e_k = p(c+s)(2q-1)$ and hence $p$ and $s$ are imperfect substitutes. The principal can still replicate the effects of an increase in $p$ (from $p$ to $xp$) by increasing the stick, but the increase in the stick must be larger than from $s$ to $xs$. More specifically, the principal will have to increase the stick from $s$ to $s' = xs + (x-1)c > xs$. In other words, the presence of the carrot requires a larger increase in the stick compared to the increase in the probability.

3. If there are carrots and not all agents fully participate: Increase *both* the carrot and the stick by multiplying it by $x$. In this case both thresholds are relevant. Increasing the carrot from $c$ to $xc$ and the stick from $s$ to $xs$ implies that their sum increases from $c + s$ to $x(c+s)$, which guarantees that $e_k$ increases by the right amount. Moreover, if both the carrot and the stick increase by the same multiplier, their ratio $\frac{c}{s}$ remains constant, which guarantees that the threshold $e_p$ increases by the right amount. Now the presence of the carrot combined with the presence of low-benefit agents who do not fully participate completely breaks the substitutability between $p$ and $s$. If the carrot is not properly adjusted, increasing $p$ and increasing $s$ may result in *opposite* effects not only on the behavior of low-benefit agents but also on the overall level of compliance.

Figure 2 below reports the results of a simple example starting from the baseline values of Figure 1. As it is easy to see, starting from equal carrot and stick, $c = s = 1$, doubling the carrot to $c' = 2$ results in a 50% increase in the *sum* of carrots and sticks (which goes from 2 to 3) and moves the compliance threshold in the same proportion, from $\frac{1}{2}$ to $\frac{3}{4}$. (Exactly the same effect can be achieved by increasing the stick from 1 to $s' = 2$.) As per Lemma 1 we can achieve the same result by increasing the probability of monitoring by 50%, from $\frac{1}{2}$ to $\frac{3}{4}$.


Figure 2: High-benefit agents' behavior, $e_k$, under a policy change (Baseline values: $c = s = 1$, $p = \frac{1}{2}$, $q = \frac{3}{4}$, $b_H = 1$, $b_L = \frac{1}{8}$; increased carrot: $c' = 2$; increased stick: $s' = 2$; increased probability: $p' = \frac{3}{4}$)

This result is completely overturned when one looks at the behavior of low-benefit agents, which is controlled by the participation threshold, $e_p$. This threshold increases in $c$, decreases in $s$ and may either increase or decrease in $p$. More precisely, $e_p$ increases in $p$ if $\frac{c}{s} > \frac{1-q}{q}$ and decreases in $p$ in the opposite case. Figure 3 shows the effects of the same policy interventions considered in Figure 2. Note that not only is the baseline behavior of low-benefit agents different from that of high-benefit agents—because the $e_p$ differs from $e_k$—but also their reactions to the same policy changes diverge. Low-benefit agents react differently to different policies and, in some cases, they react in the opposite way compared to high-benefit agents. If there are enough low-benefit agents in the population, this effect will dominate the increased compliance in the high-benefit group.

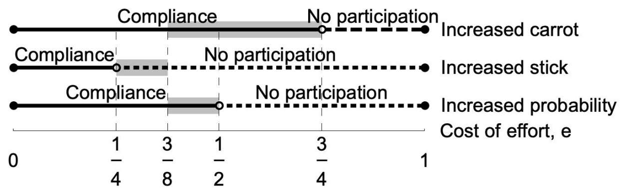


Figure 3: Low-benefit agents' behavior, $e_p$, under a policy change (Baseline values: $c = s = 1$, $p = \frac{1}{2}$, $q = \frac{3}{4}$, $b_H = 1$, $b_L = \frac{1}{8}$; increased carrot: $c' = 2$; increased stick: $s' = 2$; increased probability: $p' = \frac{3}{4}$)

More specifically, if in the baseline policy $\frac{c}{s} > \frac{1-q}{q}$, that is, if the carrot is high enough, then the threshold $e_p$ decreases in $s$ but increases in $p$, that is, $p$ and $s$ are in fact imperfect *complements* with respect to the behavior of the low-benefit agents. This is a necessary condition for an increase in $p$ and an increase in $s$ to cause opposite effects. This is however not a sufficient condition because $p$ and $s$ are always (imperfect) substitutes with respect to $e_k$, which governs the behavior of high-benefit agents. Therefore, complementarity dominates substitutability only if there are enough low-benefit agents in the population.

A striking consequence of these results is that an increase in $p$ and in $s$ may have *opposite* effects on compliance, violations and participation. To see why, consider the following setup: The population is made up of two types of agents: a portion $h = \frac{1}{4}$ of the population of agents derive a benefit $b_H = 1$ from participation, while the remaining portion $1 - h = \frac{3}{4}$ of the population derive a benefits $b_L = -1$. The accuracy of enforcement is $q_k = q_v = \frac{3}{5}$ and the carrot is $c = \frac{13}{4}$.

In the baseline, the probability of monitoring is $p = \frac{2}{3}$ and the stick is $s = \frac{1}{2}$. It is easy to check that this setting leads all high-benefit agents to participate and that the compliance threshold is $e_k = \frac{1}{2}$, which in turn implies that half of them will comply and the rest will violate the rule. In contrast, the low-benefit agents follow the participation threshold $e_p = \frac{1}{6}$, which implies that $\frac{1}{6}$ of them will participate and comply, while the remaining $\frac{5}{6}$ will refrain from participating. Overall, in this baseline setting, total participation is equal to $h + (1-h)\,e_p = \frac{3}{8}$, total compliance is $he_k + (1-h)\,e_p = \frac{1}{4}$ and the total number of violations is $h\,(1-e_k) = \frac{1}{8}$. Against this baseline we will evaluate the effects of increasing the probability $p$ or the stick $s$ by 50%.

Increasing the probability by 50% from $p = \frac{2}{3}$ to $p' = 1$ increases the benefits of participation for both types of agents. In particular, now the low-benefit agents earn a positive payoff from participating irrespective of whether they comply or violate and hence their behavior is governed by the compliance threshold $e_k$, which is now equal to $e'_k = \frac{3}{4}$. (In other words, the increase in $p$ makes the new policy violate Assumption 4, which governs the baseline policy.) Participation is now universal, that is, equal to 1, compliance is equal to $\frac{3}{4}$ and violations are equal to $\frac{1}{4}$. Compared with the baseline scenario, increasing the probability of monitoring results in more participation—up to 1 from $\frac{3}{8}$—more compliance—up to $\frac{3}{4}$ from $\frac{1}{4}$—and more violations—up to $\frac{1}{4}$ from $\frac{1}{8}$.

In contrast, increasing the magnitude of the stick by 50% from $s = \frac{1}{2}$ to $s' = \frac{3}{4}$ results in opposite results. Low-benefit agents do not fully participate, as in the baseline scenario, but their behavior is now controlled by $e'_p = \frac{1}{10}$, which is less than in the baseline scenario; similarly, high-benefit agents fully participate, as in the baseline scenario but now their behavior is controlled by $e'_k = \frac{8}{15}$, which is greater than in the baseline scenario. Therefore, compared to the baseline, increasing the magnitude of the sanction results in less participation—down to $\frac{13}{40}$ from $\frac{3}{8} = \frac{15}{40}$—less compliance—down to $\frac{5}{24}$ from $\frac{1}{4} = \frac{6}{24}$—and fewer violations—down to $\frac{7}{60}$ from $\frac{1}{8} = \frac{7.5}{60}$.

## 3 Discussion

TO COME

## 4 Conclusion

TO COME

## References


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# A Appendix

## A.1 General model

In this Appendix we prove that the results presented in the main text of the paper hold generally in the model setup presented in Section (2.1), that is, also if we relax all of the simplifying assumptions made in Section (2.2).

| Action | Payoff |
|---|---|
| The agent participates and complies | $\Pi_k(b,e) = b + pq_k c - p(1-q_k)s - e + (1-p)(\phi_c c - \phi_s s)$ |
| The agent participates but violates | $\Pi_v(b) = b + p(1-q_v)c - pq_v s + (1-p)(\phi_c c - \phi_s s)$ |
| The agent does not participate | 0 |

Table 2: Agent's payoffs with imperfect monitoring

From the payoffs reported in Table 2, the three relevant thresholds are:

$$
\begin{aligned}
b_v &\equiv p(q_v s - (1-q_v)c) - (1-p)(\phi_c c - \phi_s s) \\
e_k &\equiv p(c+s)(q_k + q_v - 1) \\
e_p(b) &\equiv b + p(q_k c - (1-q_k)s) + (1-p)(\phi_c c - \phi_s s)
\end{aligned}
$$

Where $b_v$ is such that $\Pi_v(b_v) = 0$ so that agents with $b \leq b_v$ are low-benefit agents and may elect not to participate depending on their effort cost $e$, and agents with $b > b_v$ are high-benefit agents and fully participate. Note that now we consider a continuum of agents with varying benefits from participation—not just two types—and the threshold $b_v$ partitions the population in two groups. For each of these two groups we define one additional threshold on the effort level $e$. Accordingly, $e_k$ is such that $\Pi_k(b, e_k) = \Pi_v(b)$ so that, conditional on $b > b_v$, that is conditional on participation, high-benefit agents with $e < e_k$ comply and those with $e \geq e_k$ violate. Finally, $e_p(b)$ is such that $\Pi_k(e_p(b), b) = 0$ so that, conditional on $b \leq b_v$, low-benefit agents with $e < e_p$ comply and those with $e \geq e_p$ stay out of the regulated activity. Figure 4 helps visualizing this general setup.

Note the comparative statics reported in Table 3, which will be useful in the analysis.

| Policy variable | $b_v$ | $e_k$ | $e_p(b)$ |
|---|---|---|---|
| $c$ | Decrease | Increase | Increase |
| $s$ | Increase | Increase | Decrease |
| $p$ | Ambiguous | Increase | Ambiguous |

Table 3: Comparative statics

## A.2 Case with $\phi_c = \phi_s = 0$.

We start with the simple case in which carrots and sticks are applied only to the audited agents, that is, $\phi_c = \phi_s = 0$. The thresholds reduce to:

$$
\begin{aligned}
b_v &= p(q_v s - (1-q_v)c) \\
e_k &= p(c+s)(q_k + q_v - 1) \\
e_p(b) &= b + p(q_k c - (1-q_k)s)
\end{aligned}
$$

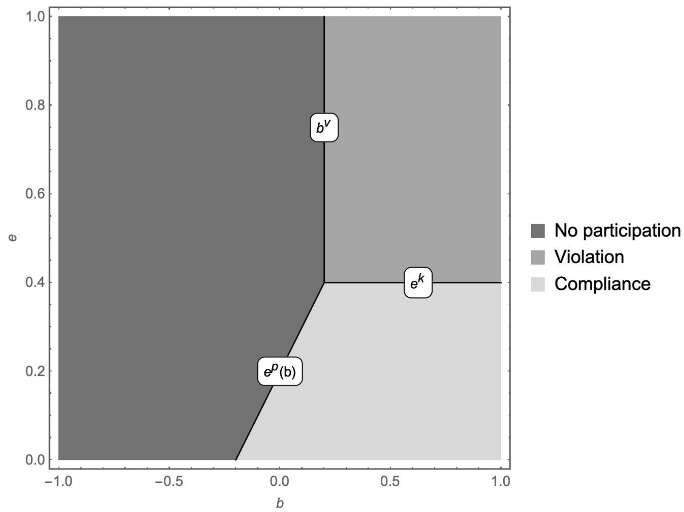



Figure 4: A baseline case: $c = 2$, $s = 2$, $p = \frac{1}{2}$, $q_k = q_v = \frac{3}{5}$

We prove the following three propositions.

**Proposition 1.** *When $\phi_c = \phi_s = 0$, if there is no carrot ($c = 0$) and there is universal participation ($b > b_v$ for all $b$), then $p$ and $s$ are perfect substitutes, that is, increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have the same effects on compliance, participation and violations. Thus, to replicate the effects of an increase in the probability from $p$ to $xp$, the principal should increase the stick from $s$ to $xs$.*

*Proof.* If $b > b_v$ for all $b$, then all agents fully participate and the only relevant threshold on $e$ is $e_k$. Note that if $c = 0$, then $e_k = ps(q_k + q_v - 1)$. Visual inspection confirms the result. □

**Proposition 2.** *When $\phi_c = \phi_s = 0$, if there are carrots ($c > 0$) and there is universal participation ($b > b_v$ for all $b$), then $p$ and $s$ are imperfect substitutes, that is, increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have qualitatively similar effects on compliance, participation and violations, but the magnitudes of these effects are different. To replicate the effects of an increase in the probability from $p$ to $xp$, the principal should, alternatively:*

- *increase the sum of carrots and sticks from $c + s$ to $x(c + s)$, if the principal can adjust carrots and sticks, or*
- *increase the stick from $s$ to $xs + (x - 1)c > xs$, if the principal can adjust only the stick.*

*Proof.* If $b > b_v$ for all $b$, then all agents fully participate and the only relevant threshold is $e_k$. It is evident that $p$ and the sum $c + s$ are perfect substitutes, which proves the first point. To prove the second point note that the principal must increase the stick from $s$ to $s'$ such that

$$xp(c + s)(q_k + q_v - 1) = p(c + s')(q_k + q_v - 1)$$

which reduces to $x(c + s) = c + s'$ or $s' = x(c + s) - c = xs + (x - 1)c$. □

**Proposition 3.** *When $\phi_c = \phi_s = 0$, if there are carrots ($c > 0$) and there is not universal participation ($b \leq b_v$ for some $b$) then:*

*1. If $\frac{c}{s} < \frac{1-q_k}{q_k} < \frac{q_v}{1-q_v}$, then p and s are imperfect substitutes; increasing p and increasing s by the same multiplier $x > 1$ have qualitatively similar effects on violations and participation (but the magnitudes of these effects are different) but may have opposite effects on compliance.*

*2. If $\frac{1-q_k}{q_k} \leq \frac{c}{s} < \frac{q_v}{1-q_v}$, then p and s are imperfect substitutes for the high-benefit agents (those with $b > b_v$), but are imperfect complements for the low-benefit agents (those with $b \leq b_v$); increasing p and increasing s by the same multiplier $x > 1$ have qualitatively similar effects on violations (but the magnitudes of these effect are different) but may have opposite effects on compliance and participation.*

*3. If $\frac{c}{s} \geq \frac{q_v}{1-q_v} > \frac{1-q_k}{q_k}$, then p and s are imperfect substitutes for the high-benefit agents (those with $b > b_v$), but are imperfect complements for the low-benefit agents (those with $b \leq b_v$); increasing p and increasing s by the same multiplier $x > 1$ may have opposite effects on compliance and violations, and surely have opposite effects on participation.*

*To replicate the effect of an increase in the probability from p to xp, the principal should increase both the stick from s to xs and the carrot from c to xc; the principal cannot replicate the effect of an increase in probability by simply adjusting the stick.*

*Proof.* If $b \leq b_v$ for some $b$, then all three thresholds are relevant. First note that if the principal multiplies both the stick and the carrot by $x$ we have

$$\begin{aligned} b_v &= p\left(q_v xs - (1-q_v)\, xc\right) &&= xp\left(q_v s - (1-q_v)\, c\right) \\ e_k &= p\left(xc + xs\right)\left(q_k + q_v - 1\right) &&= xp\left(c+s\right)\left(q_k + q_v - 1\right) \\ e_p\left(b\right) &= b + p\left(q_k xc - (1-q_k)\, xs\right) &&= b + xp\left(q_k c - (1-q_k)\, s\right) \end{aligned}$$

which proves the conversion rule at the end of the proposition: $c' = xc$ and $s' = xs$. The fact that an adjustment in $s$ is never sufficient follows from noting that each of the thresholds would require a different adjustment. Second, note that the comparative statics in Table 3 is still valid when $\phi_c = \phi_s = 0$ and implies that an increase in $s$ and in $p$ may have opposite effects on the thresholds $b_v$ and $e_p(b)$. In particular, note that we have $\frac{\partial b_v}{\partial s} > 0$, $\frac{\partial e_k}{\partial s} > 0$ and $\frac{\partial e_p(b)}{\partial s} < 0$. While the effect of $p$ depends on the ratio $\frac{c}{s}$ as summarized in Table 4. (Note that our assumptions imply $\frac{q_v}{1-q_v} > 1 > \frac{1-q_k}{q_k}$.)

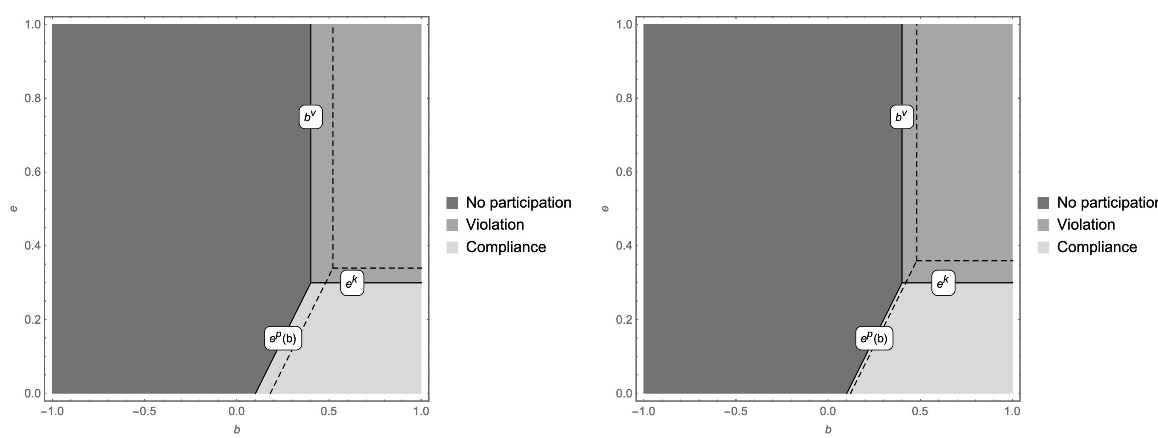
| | $\frac{\partial b_v}{\partial p}$ | $\frac{\partial e_k}{\partial p}$ | $\frac{\partial e_p(b)}{\partial p}$ |
|---|---|---|---|
| $\frac{c}{s} < \frac{1-q_k}{q_k} < \frac{q_v}{1-q_v}$ | + | + | - |
| $\frac{1-q_k}{q_k} \leq \frac{c}{s} < \frac{q_v}{1-q_v}$ | + | + | + |
| $\frac{1-q_k}{q_k} < \frac{q_v}{1-q_v} \leq \frac{c}{s}$ | - | + | + |

Table 4: Effects of increasing $p$ on the relevant thresholds

Therefore, the derivatives $\frac{\partial e_p(b)}{\partial p}$ and $\frac{\partial e_p(b)}{\partial s}$ have the same sign if $\frac{c}{s} < \frac{1-q_k}{q_k}$ and have opposite signs otherwise; similarly, the derivatives $\frac{\partial b_v}{\partial p}$ and $\frac{\partial b_v}{\partial s}$ have the same sign if $\frac{c}{s} < \frac{q_v}{1-q_v}$ and have opposite signs otherwise, which gives rise to the three cases discussed in the proposition. The following figures illustrate these three cases. All simulations are based on the baseline setting of

Figure 4, with $q_k = q_v = \frac{3}{5}$ and $s = 2$ and $p = \frac{1}{2}$ as a starting point. We compare in turn the effects or increasing the stick and increasing the probability by the same factor (20%) in three scenarios: in Figure 5 we set $c = 1$, which results in a low carrot-to-stick ratio ($\frac{c}{s} < \frac{1-q_k}{q_k} < \frac{q_v}{1-q_v}$); in Figure 6 we set $c = 2$, which results in an intermediate carrot-to-stick ratio ($\frac{1-q_k}{q_k} \leq \frac{c}{s} < \frac{q_v}{1-q_v}$); finally, in Figure 7 we set $c = 4$, which results in a high carrot-to-stick ratio ($\frac{c}{s} \geq \frac{q_v}{1-q_v} > \frac{1-q_k}{q_k}$).[7]

Figure 5 illustrates case 1) in the Proposition. When the carrot is relatively small compared to the stick, increasing $s$ and $p$ by the same amount results in qualitatively the same effects on violations (which decrease) and participation (which also decreases). The effects on violations is ambiguous because while the high-benefit agents react to an increase in the stick or the probability by complying more (rather than violating), the low-benefit agents react by participating (and hence complying) less. Depending on which of these effects prevails (which in turn depends on the distribution of $b$ and $e$), compliance may increase or decrease. Since an increase in $p$ and an increase in $s$ impact the compliance region in different ways, they may have opposite effects on compliance: it could be the case that while increasing $p$ incrreases compliance, increasing $s$ reduces it. Finally, it is easy to see that the magnitude of the effects of increased $s$ and $p$ are different.



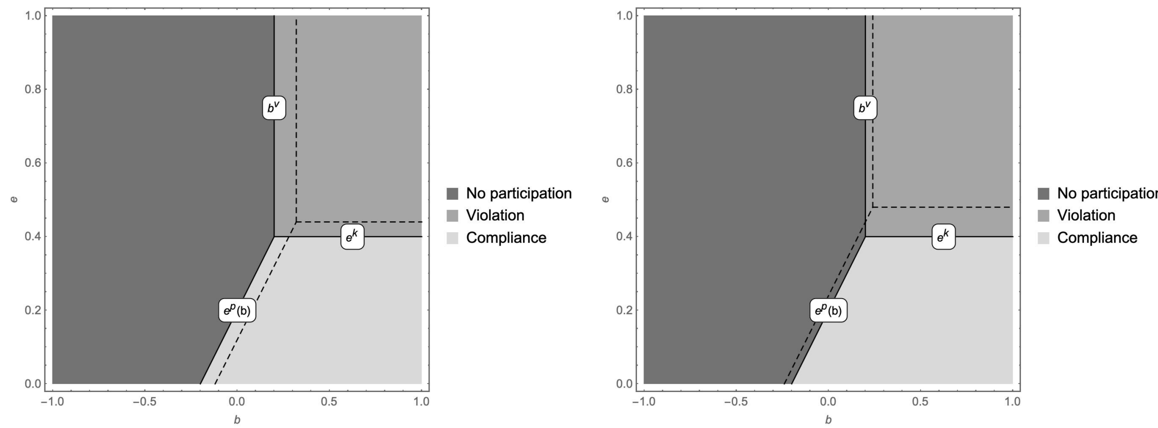

(a) Increasing the stick (by 20% from $s = 2$ to $s = \frac{12}{5}$) (b) Increasing the probability (by 20% from $p = \frac{1}{2}$ to $p' = \frac{3}{5}$)

Figure 5: Low carrot-to-stick ratio ($c = 1$)

Similarly, Figure 6 illustrates case 2) in the Proposition. Now $p$ and $s$ have opposite effects on the threshold $e_p(b)$. While in both cases violations decrease, the effects on participation and compliance may be different. While increasing $p$ unambiguously improves compliance, an increase in $s$ has ambiguous effects on compliance and hence may reduce it. Likewise, while an increase in $s$ unambiguously reduces participation, an increase in $p$ has ambiguous effects on participation and hence may increase it.

Finally, Figure 7 illustrates case 3) in the Proposition. Now $p$ and $s$ have opposite effects both on $e_p(b)$ and on $b_v$. While increasing $s$ unambiguously reduces violations, increasing $p$ may reduce or increase violations. Similarly, while increasing $p$ unambiguously increases compliance, increasing

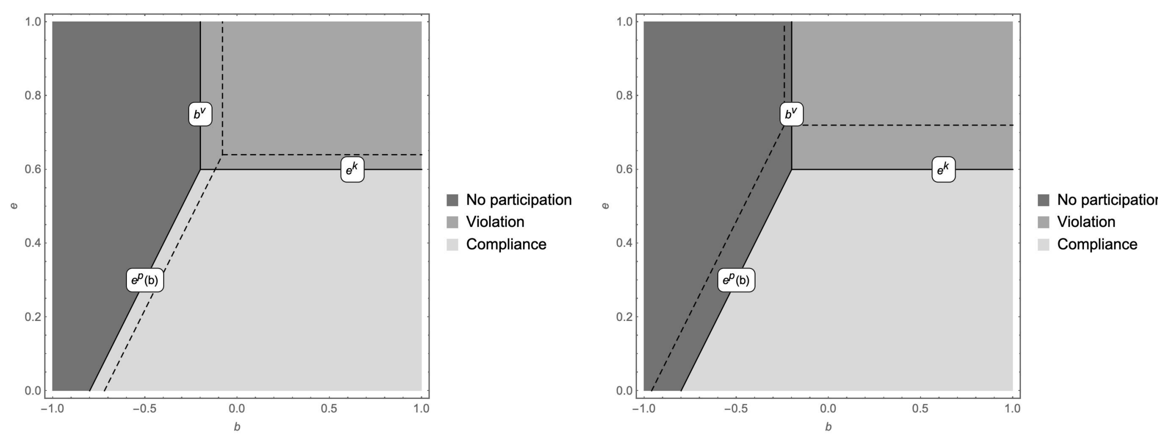
[7]Note that these conditions also hold after the stick is increased.




(a) Increasing the stick (by 20% from $s = 2$ to $s = \frac{12}{5}$) (b) Increasing the probability (by 20% from $p = \frac{1}{2}$ to $p' = \frac{3}{5}$)

Figure 6: Intermediate carrot-to-stick ratio ($c = 2$)

$s$ may increase or reduce compliance. Finally, while increasing $s$ reduces participation, increasing $p$ increases participation.




(a) Increasing the stick (by 20% from $s = 2$ to $s = \frac{12}{5}$) (b) Increasing the probability (by 20% from $p = \frac{1}{2}$ to $p' = \frac{3}{5}$)

Figure 7: High carrot-to-stick ratio ($c = 4$)

□

**Corollary 1.** *When $\phi_c = \phi_s = 0$, if there are carrots ($c > 0$), there is not universal participation ($b \leq b_v$ for some $b$) and the carrot is sufficiently larger than the stick ($\frac{c}{s} > \frac{q_v}{1-q_v} > \frac{1-q_k}{q_k}$), then an increase in $p$ and in $s$ may have opposite effects on compliance, violations and participation.*

*Proof.* This case fits Proposition 3, point 3. To prove the corollary, it is enough to show that there are parameter values producing the result. See the main text in Section 2.4. □

### A.3 Case with $\phi_c = 1$ and $\phi_s = 0$.

Let us now turn to a realistic sub-case in which non-monitored agents receive a carrot with certainty, that is, where $\phi_c = 1$ and $\phi_s = 0$. In this case, we have

$$\begin{aligned} b_v &= pq_v (c+s) - c \\ e_k &= p(c+s)(q_k + q_v - 1) \\ e_p(b) &= b - p(1-q_k)(c+s) + c \end{aligned}$$

Here the result provided in Proposition 1 and 2 hold without the restriction of universal participation. In general, in this case, $p$ and $s$ have the same qualitative effects.

**Proposition 4.** *When $\phi_c = 1$ and $\phi_s = 0$, if there is no carrot ($c = 0$), then $p$ and $s$ are perfect substitutes, that is, increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have the same effects on compliance, participation and violations. To replicate the effects of an increase in the probability from $p$ to $xp$, the principal should increase the stick from $s$ to $xs$.*

*Proof.* The proof is similar to the proof of Proposition 1 and is omitted. □

**Proposition 5.** *When $\phi_c = 1$ and $\phi_s = 0$, if there are carrots ($c > 0$), then $p$ and $s$ are imperfect substitutes, that is, increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have qualitatively similar effects on compliance, participation and violations, but the magnitudes of these effects are different. To replicate the effects of an increase in the probability from $p$ to $xp$, the principal should, alternatively:*

- *increase the sum of carrots and sticks from $c + s$ to $x(c + s)$, if the principal can adjust carrots and sticks, or*
- *increase the stick from $s$ to $xs + (x - 1)c > xs$, if the principal can adjust only the stick.*

*Proof.* The proof is similar to the proof of Proposition 2 and is omitted. □

Differently from the case of the previous section, when carrots are given to non-monitored agents, that is, when they are not conditional on being monitored, there is a weaker link between the probability of monitoring and the carrot and $p$ and $s$ remain (imperfect) substitutes even when there is not universal participation. The intuition is that with now the carrot behaves more like a stick: the only case in which monitoring is relevant is when the agent is monitored and found violating, which is when the carrot is denied. But note that this is also the case in which the stick is applied. In all other cases, that is when the agent is monitored and found in compliance and when the agent is not monitored, there is no stick and no denial of the carrot. Hence, what matters for incentive purposes in this setting is the sum of carrots and stick times the probability of monitoring, as it is evident from the functional form of the three thresholds in this case, as indicated above.

## A.4 General case.

Finally, we consider the general case, that is, when $0 < \phi_c < 1$ and $0 < \phi_s < 1$. Note that in this case the relevant thresholds are

$$\begin{aligned} b_v &= p\left(q_v s - (1-q_v)\,c\right) - (1-p)\left(\phi_c c - \phi_s s\right) \\ e_k &= p\,(c+s)\,(q_k + q_v - 1) \\ e_p\,(b) &= b + p\left(q_k c - (1-q_k)\,s\right) + (1-p)\left(\phi_c c - \phi_s s\right) \end{aligned}$$

**Proposition 6.** *In the general case, if there is no carrot ($c = 0$) and there is universal participation ($b > b_v$ for all $b$), then $p$ and $s$ are perfect substitutes, that is, increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have the same effects on compliance, participation and violations. To replicate the effects of an increase in the probability from $p$ to $xp$, the principal should increase the stick from $s$ to $xs$.*

*Proof.* The proof is similar to the proof of Proposition 1 and is omitted. □

**Proposition 7.** *In the general case, if there are carrots ($c > 0$) and there is universal participation ($b > b_v$ for all $b$), then $p$ and $s$ are imperfect substitutes, that is, increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have qualitatively similar effects on compliance, participation and violations, but the magnitudes of these effects are different. To replicate the effects of an increase in the probability from $p$ to $xp$, the principal should, alternatively:*

- *increase the sum of carrots and sticks from $c + s$ to $x\,(c+s)$, if the principal can adjust carrots and sticks, or*
- *increase the stick from $s$ to $xs + (x-1)\,c > xs$, if the principal can adjust only the stick.*

*Proof.* The proof is similar to the proof of Proposition 2 and is omitted. □

**Proposition 8.** *In the general case, if there are carrots ($c > 0$) and there is not universal participation ($b \le b_v$ for some $b$) then:*

1. *If $\frac{c}{s} < \frac{1-q_k-\phi_c}{q_k+\phi_s} < \frac{q_v-\phi_c}{1-q_v+\phi_s}$, then $p$ and $s$ are imperfect substitutes; increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have qualitatively similar effects on violations and participation (but the magnitudes of these effect is different) but may have opposite effects on compliance.*

2. *If $\frac{1-q_k-\phi_c}{q_k+\phi_s} \le \frac{c}{s} < \frac{q_v-\phi_c}{1-q_v+\phi_s}$, then $p$ and $s$ are imperfect substitutes for the high-benefit agents (those with $b > b_v$), but are imperfect complements for the low-benefit agents (those with $b \le b_v$); increasing $p$ and increasing $s$ by the same multiplier $x > 1$ have qualitatively similar effects on violations (but the magnitudes of these effect are different) but may have opposite effects on compliance and participation.*

3. *If $\frac{c}{s} \ge \frac{q_v-\phi_c}{1-q_v+\phi_s} > \frac{1-q_k-\phi_c}{q_k+\phi_s}$, then $p$ and $s$ are imperfect substitutes for the high-benefit agents (those with $b > b_v$), but are imperfect complements for the low-benefit agents (those with $b \le b_v$); increasing $p$ and increasing $s$ by the same multiplier $x > 1$ may have opposite effects on compliance and violations, and surely have opposite effects on participation.*

*To replicate the effect of an increase in the probability from $p$ to $xp$, the principal should increase both the stick from $s$ to $xs$ and the carrot from $c$ to $xc$; the principal cannot replicate the effects of an increase in probability by simply adjusting the stick.*

*Proof.* The proof is similar to the proof of Proposition 3 and is omitted. □